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A STUDY ON FUZZY RELIABILITY FOR WEB SERVER USING STATISTICAL APPROACH

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ABSTRACT

In this paper the method for finding the reliability of the web systembased on fuzzy probabilistic semi-Markov chain that are based on uncertainty/vagueness has been discussed. The results for the fuzzy model are provided by means of the fuzzy probabilities and are modeled by triangular fuzzy numbers. The statistical data approachhas been applied for considering uncertainty in the fuzzy probability. The method is also explained using an illustration.

KEYWORDS: Fuzzy Probabilistic Semi-Markov Model, Non- Homogenous Transition Fuzzy Possibility Matrix, Fuzzy Reliability, Statistical Confidence Interval

INTRODUCTION

In this paper the web system are as a non-homogenous fuzzy markov model, assigned in order together with transition fuzzy probability between them. In conventional reliability theory, there are two fundamental hypothesis (1) The binary state assumption, (2) Probability state assumption. However considering the incomplete or non-obtainable information in web system problems, when the usual conventional reliability analysis is inadequate to calculate such uncertainties in data. The collected data or system parameters are often fuzzified as referred in papers by Cai, Wen and Zhang[2-4].The fuzzy state assumption: At any given time, the system has only two states. One is the fuzzy success state and another is the fuzzy failure state. They used alpha cut of triangular fuzzy number to get the interval and find the fuzzy reliability of the serial and the parallel system. Same methodology have been studied by Prabha [6] for finding the reliability of the web system. They did not initially apply a statistical methodology. Chen [7], and Mon[5] likewise omitted the statistical application. Jing-Shing Yao [1] used statistical method for finding the reliability of serial and parallel using the statistical confidence interval instead of the point estimate. A similar method has been employed in this paper for finding the fuzzy reliability of the system.

PRELIMINARIES

Fuzzy Numbers

A fuzzy number A is a fuzzy set which is defined on a real number system R with membership function

- $\mu_A: R \longrightarrow [0, 1]$ and it passes the following properties
- A must be convex
- A must be normal
- It membership function must be piecewise continuous

Triangular Fuzzy Number

A triangular fuzzy number A can be defined a triplet (a_1, a_2, a_3) where $a_1 \le a_2 \le a_3$ and its membership function is

$$\mu_A(x) = \begin{cases} 0 \; if \; x < a1 \\ \frac{x-a1}{a2-a1} \; if \; a1 < x < a2 \\ \frac{a3-x}{a3-a2}, if \; a2 < x < a3 \\ 0, otherwise \end{cases}$$

Fuzzy Reliability

The fuzzy reliability of the proposed system is defined to be the fuzzy probability that the system eventually completes its task successfully without failing from transition from one state to other state till its termination state is reached.

FUZZY PROBABILISTIC ANALYSIS OF NON HOMOGENEOUS SEMI- MARKOV MODEL

In Web modeling even though the outcomes is certain there exist uncertainty in the probability of events and these uncertainties in the probabilistic usage information are represented by fuzzifying the probability values into a Triangular fuzzy number on [0, 1] for the system to perform its function properly. A Non- homogenous fuzzy possibilitstic Markov Chain which has finite number of states is a Markov Chain whose transition fuzzy possibilities, from state i to state j from time step n to n+1 is given by

$$\breve{P}_{ij}(n, n+1) = \tilde{\sigma}(\tilde{f}_{n+1} = j/\tilde{f}_n = i); \; n \in N, \; i,j \in S \tag{1}$$

are the function of n. These transition fuzzy possibility values form a square fuzzy matrix $\breve{P}_{(nxn)}(n, n+1)$ whose entries are triangular fuzzy numbers on [0,1] and $\breve{P}_{(nxn)}(n, n+1)$ is called the non- homogenous transition fuzzy possibility matrix.

$$\breve{P}_{ij}(n, n+1) = \begin{bmatrix} \tilde{f}_{11}(n, n+1) & \cdots & \tilde{f}_{1n}(n, n+1 \\ \vdots & \ddots & \vdots \\ \tilde{f}_{n1}(n, n+1 & \cdots & \tilde{f}_{nn}(n, n+1 \end{bmatrix} \tag{2}$$

Hence the transition fuzzy possibility matrix $\breve{P}_{ij}$ varies with respect time (n), with

$$\max_j\{\breve{P}ij(n, n+1)\} = (1,1,1) \tag{3}$$

The transition fuzzy possibilities $\breve{P}_{ij}$ are called transition fuzzy possibility function of the non- homogenous fuzzy possibilitic markov chain. It is also called one – step transition fuzzy possibility function and its rows are fuzzy possibility distribution. Again the multistep transition fuzzy possibility fuction can be written as follows;

$$\breve{P}_{ij}(m,n) = \tilde{\sigma}(\tilde{f}_n = j/\tilde{f}_m = i); \; 0 \le m \le n \tag{4}$$

The intitial fuzzy possibility vector $\tilde{P}^0$ on the state space S is denote as $\tilde{P}^0 = (\tilde{P}_1^{(0)}, \tilde{P}_2^{(0)} \dots, \tilde{P}_n^{(0)})$ where $\tilde{P}_1^{(0)}$ is the fuzzy possibility of being in state i initially and it is defined as

$$\tilde{P}_1^{(0)} = \tilde{\sigma}(\tilde{f}_0 = i) \tag{5}$$

FUZZY RELIABILITY MODEL

Fuzzy reliability is a concept in which fuzzy sets can capture subjective, uncertain and ambiguous information in a system. Now we present the fuzzy reliability modeling using fuzzy probabilistic Semi-Markov model based on the fuzzy profust reliability theory through the transition fuzzy probabilities expressed as triangular numbers. The triangular numbers are determined using confidence interval obtained from the statistical datas.

Consider a fuzzy probabilistic Semi-Markov model $\{(S_n, T_n), n \in N\}$ consisting of 'n' states together with transition time. Let $U = \{s_1, s_2, ..., s_n\}$ denote the universe of discourse.

On this universe we define, a fuzzy success state S in the time interval [t0, t0+t]

$$S = \{S_i, \tilde{\mu}_S(S_i)[\, t_0, t_0 + t];\, i = 1,2, \dots, n\} \tag{6}$$

and a fuzzy failure state F:

$$F = \{S_i, \tilde{\mu}_F(S_i)[\, t_0, t_0 + t];\, i = 1,2, \dots, n\} \tag{7}$$

Where $\tilde{\mu}_S(S_i)[\, t_0, t_0 + t]$ and $\tilde{\mu}_F(S_i)[\, t_0, t_0 + t]$ are triangular fuzzy numbers represented by $\tilde{\mu}_S(S_i)[\, t_0, t_0 + t]$ *and* $\tilde{\mu}_F(S_i)[\, t_0, t_0 + t]$ are triangular number, which are determined as follows $[\tilde{s}_{si} - t_{n_i-1}(\gamma_1)(\sigma_j/\sqrt{n_i}, \tilde{s}_{si}, \tilde{s}_{si} + t_{n_i-1}(\gamma_2)(\sigma_j/\sqrt{n_i})]$ & $[\tilde{s}_{Fi} - t_{n_i-1}(\gamma_1)(\sigma_j/\sqrt{n_i}, \tilde{s}_{Fi}, \tilde{s}_{Fi} + t_{n_i-1}(\gamma_2)(\sigma_j/\sqrt{n_i})]$.

From the data collected we estimate the success rate and failure rate, then we can estimate the (1- γ)100% confidence interval for the success rate and failure rate. The confident limits are taken as the upper and lower limit of the triangular number the success rate and failure rate is taken as the middle value of the triangular number respectively.

A fuzzy state is just a fuzzy set and fuzzy states represent the system level of performance. It is noted that when fuzziness of interest is discarded, the fuzzy success state and the fuzzy failure state become a conventional success and failure state respectively. In the conventional reliability theory, the reliability of the system is determined based on the event transition from system success state to system failure state. Accordingly we are here interested in the event denoted by HSF of transition from the fuzzy success state to the fuzzy failure state. we define

$$\tilde{R}[\, t_0 , t_0 + t] = \tilde{\sigma}[\tilde{H}_{SF} \textit{ does not occur in the time interval } [\, t_0 , t_0 + t]] \tag{8}$$

$\tilde{R}[\, t_0 , t_0 + t]$ is referred as the fuzzy interval reliability of the system in the time interval $[\, t_0 , t_0 + t]$

To compute the fuzzy interval reliability we must express $\tilde{H}_{SF}$. Since both S and F are fuzzy states, the transition between them are consequently fuzzy. We view as a fuzzy event. Apparently $\tilde{H}_{SF}$ may occur only when some state transition occur among of n system states

$\{s_1, s_2, .., s_n\}$, So $\tilde{H}_{SF}$ can be defined on the universe $\tilde{U}_T = \{\tilde{P}ij[t_0 , t_0 + t], i, j = 1,2, \dots, n\}$, where $\tilde{P}ij\, [t_0 , t_0 + t]$ represents the transition fuzzy probability from Si to Sj with membership function: $\{\tilde{\mu}_{\tilde{H}_{SF}}\ (\tilde{P}ij[t_0 , t_0 + t]);\ i,j = 1,2,3\dots n\ \}$ represented as

$$\tilde{\mu}_{\tilde{H}_{SF}}\ (\tilde{P}ij[t_0 , t_0 + t] = (\tilde{\mu}^1{}_{\tilde{H}_{SF}}\ (\tilde{P}ij[t_0 , t_0 + t]), \tilde{\mu}^2{}_{\tilde{H}_{SF}}\ (\tilde{P}ij[t_0 , t_0 + t]), \tilde{\mu}^3{}_{\tilde{H}_{SF}}\ (\tilde{P}ij[t_0 , t_0 + t]))\ .$$

Let $\beta^k_{F/S}(S_i)[t_0\,,t_0+t] = \frac{\mu^k_F(S_i)[t_0\,,t_0+t]}{\mu^k_F(S_i)[t_0\,,t_0+t]+\mu^k_S(S_i)[t_0\,,t_0+t]}$) (9)

Then $\beta^k_{F/S}(S_i)[t_0\,,t_0+t]$ can be viewed as the grade of membership of S irelative to S_j.

F. It is reasonable to say that the fuzzy transition from Si to Sj makes the fuzzy transition from S to F occurs to some extent if and only if the relation

($\beta^k_{F/S}(S_j)[t_0\,,t_0+t]$) > ($\beta^k_{F/S}(S_i)[t_0\,,t_0+t]$) holds:

We therefore define

$$\tilde{\mu}_{\tilde{H}_{SF}}(\tilde{P}ij[t_0\,,t_0+t] = \begin{cases} \beta_{F/S}(S_j)[t_0\,,t_0+t]-\beta_{F/S}(S_i)[t_0\,,t_0+t],\ for \\ \qquad \beta_{F/S}(S_j)[t_0\,,t_0+t] > \beta_{F/S}(S_j)[t_0\,,t_0+t] \\ 0,\ for\ \beta_{F/S}(S_j)[t_0\,,t_0+t] \le \beta_{F/S}(S_j)[t_0\,,t_0+t] \end{cases} \qquad (10)$$

Hence the fuzzy interval reliability can be expressed as

$\tilde{R}$[t_0 , t_0 + t] = $\tilde{\sigma}[\tilde{H}_{SF}\ does\ not\ occur\ in\ the\ time\ interval\ [\,t_0\,,t_0+t]]$, which is a triangular fuzzy number $\tilde{R}$[t_0 , t_0 + t] =$\tilde{R}^k([\,t_0\,,t_0+t])$, for k = 1,2,3 is defined by

$\tilde{R}^k([\,t_0\,,t_0+t]) = 1 - \sum_{i=1}^{n}\sum_{j=1}^{n}\tilde{\mu}_{\tilde{H}_{SF}}(\tilde{P}ij[t_0\,,t_0+t]).(\tilde{\pi}ij[t_0\,,t_0+t])$, k=1, 2, 3.

Note that $\tilde{\pi}ij[t_0\,,t_0+t]$ is the complement of $\tilde{P}ij^k[t_0\,,t_0+t]$, for i,j = 1 to n. Thus the the fuzzy reliability of the system at is given by $\tilde{R}$(t).

ILLUSTRATION

In the following, the above defined fuzzy reliability model is explained with an example. Suppose that there exist 3 web pages (states) A, B, C and transition from one page to another web page of a particular website for a time period of ten days. Since there exists uncertainties in the probabilistic usage information between the state transitions, for each transition we associate fuzzy transition defined as transition fuzzy probabilities obtained as follows: N be the total number of transitions from state i to state j and s-the number of successes among them, then the ratio for transition probability is *N/s*. Since these values (N and s) which are extracted from the path that exists in the specified period of time are not exact, we fuzzify these values by finding the (1- γ)100% confidence interval(1- γ)100% confidence interval for the success rate (i.e) $[\bar{s}_i - t_{n_i-1}(\gamma_1)(\sigma_j/\sqrt{n_i}, \bar{s}_i, \bar{s}_i + t_{n_i-1}(\gamma_2)(\sigma_j/\sqrt{n_i})]$, where $\bar{S}i$ and $\tilde{\sigma}i$ are the average and standard deviation of the success rate. As the access of web increases, the data for the access will also increase. Hence we have modeled fuzzy probabilistic semi-Markov model with state space U= {A, B, C} and transitions as transition fuzzy probabilities and is depicted below.

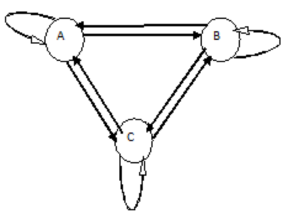

Figure 1

The fuzzy success state, fuzzy failure state and the transition fuzzy probabilities for the given state space are interpreted as follows:

Success State: (A/(.91336,.92199,.92930), B/(.85726,.87017,.88272), C/(.85683,.8656,.87403))

Failure State: (A/ (.0707,.07867,.08664), B/(.12134,.13197,.14239), C/(.12597,.13446,.14318))

$$TPM = \begin{pmatrix} (.91336,.92133,.92930) & (.90101,.91219,.92307) & (.89222,.90682,.92102) \\ (.85726,.86804,.87866) & (.85726,.87017,.88272) & (.85504,.86453,.87376) \\ (.86282,.87766,.89208) & (.90483,.91345,.92182) & (.85683,.86555,.87403) \end{pmatrix} \quad (11)$$

The grade membership function from success to failure are as follows:

$$\beta_{F/S}(A)[t_0, t_0 + t] = (.07184, .07867, .08528)$$

$$\beta_{F/S}(B)[t_0, t_0 + t] = (.12401, .131965, .13988)$$

$$\beta_{F/S}(C)[t_0, t_0 + t] = (.12817, .134455, .140753)$$

The calculate fuzzy transition from state I to state j making the fuzzy transition from S to F occurs as follow

$$\tilde{\mu}_{R_{SF}}(\tilde{P}\text{ij}[0,10]) = \begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{pmatrix} (0,0,0) & (.05217,.053295,.0546) & (.05633,.055785,.055473) \\ (0,0,0) & (0,0,0) & (.00416,.002485,.000873) \\ (0,0,0) & (0,0,0) & (0,0,0) \end{pmatrix} \end{array}$$

Reliability = 1-(.00890976,.010125,.01132522)

= (.98867,.98987,.99109)

It is observe that the fuzzy reliability is reasonable for the fuzzy success and fuzzy failure.

CONCLUSIONS

A new method for finding fuzzy system reliability using fuzzy profust reliability theory as been discussed and obtained the results as a triangular fuzzy number. Since in most of the real life system, the system states are very often fuzzy, moreover the statisitical method of fuzzyfying the data give more accurate result and the above constructed method can be applied for all the models that are modeled as a fuzzy probabilistic semi-Markov model.

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